

Scattering states in the four nucleons system

F. Ciesielski, J. Carbonell, C.Gignoux ^a

^aInstitut des Sciences Nucléaires, 53 Av. des martyrs, 38026 Grenoble, France

The Faddeev-Yakubowski equations have been solved in configuration space for the four nucleons system. Results for bound and scattering states in the isospin and S-wave approximation for different (T,S) channels are presented. The n-t elastic cross section has been also calculated with realistic interactions. A special interest was devoted to the description of the observed resonant structure at $T_{lab} \approx 3$ MeV which is well reproduced only by the simple MT I-III potential.

1. INTRODUCTION

We present in this contribution the results obtained by solving the Faddeev-Yakubowski (FY) equations in configuration space for the 4N system with special interest in their scattering states. The A=4 chart presents a very rich continuum and constitutes a challenge for the theoretical models. We have limited our study to the energy region below the first inelastic channel. All the calculations have been carried out in the isospin approximation, i.e neglecting Coulomb forces. Next section will be devoted to a short presentation of the FY formalism. In Section 3 we will discuss the results: first those obtained in the S-wave approximation for the different (T,S) channels and secondly those concerning the n-t scattering with realistic interactions.

2. FORMALISM

The FY equations for 4 interacting particles are obtained by first splitting the total wavefunction Ψ in the usual Faddeev amplitudes, Ψ_{ij} , associated with each of the interacting pair. Each of these amplitudes is in its turn splitted in 3 parts, the FY amplitudes, corresponding to the different asymptotics of the remaining two non interacting particles. The Schrödinger equation is equivalent to the set of 18 coupled equations for which it is possible to define appropriate boundary conditions ensuring the unicity of the solution.

In the case of identical particles, the 18 FY amplitudes can be obtained by the action of the permutation operators P_{ij} on two of them, denoted K and H . The $K - H$ amplitudes satisfy the following equations:

$$(E - H_0 - V)K = V[(P_{23} + P_{13})(\varepsilon + P_{34})K + \varepsilon(P_{23} + P_{13})H] \quad (1)$$

$$(E - H_0 - V)H = V[(P_{13}P_{24} + P_{14}P_{23})K + P_{13}P_{24}H] \quad (2)$$

in which $\varepsilon = \pm 1$ depending on whether we are dealing with bosons or fermions. Each amplitude $F = K, H$ is considered as function of its own set of Jacobi coordinates $\vec{x}, \vec{y}, \vec{z}$

and expanded in angular momentum variables for each coordinate according to

$$\langle \vec{x}\vec{y}\vec{z} | F \rangle = \sum_{\alpha} \frac{F_{\alpha}(xyz)}{xyz} Y_{\alpha}(\hat{x}, \hat{y}, \hat{z}) \quad (3)$$

where Y_{α} are generalized tripolar harmonics containing spin, isospin and angular momentum variables and the functions F_{α} , called FY components, are the unknowns. The label α holds for the set of intermediate quantum numbers defined in a given coupling scheme and includes the specification for the type of amplitudes K or H. We have used the following couplings:

$$\begin{aligned} \text{K amplitudes} & \quad \left\{ [(t_1 t_2)_{\tau_x} t_3]_{T_3} t_4 \right\}_T \otimes \left\{ \left[(l_x (s_1 s_2)_{\sigma_x})_{j_x} (l_y s_3)_{j_y} \right]_{J_3} (l_z s_4)_{j_z} \right\}_J \\ \text{H amplitudes} & \quad \left[(t_1 t_2)_{\tau_x} (t_3 t_4)_{\tau_y} \right]_T \otimes \left\{ \left[(l_x (s_1 s_2)_{\sigma_x})_{j_x} (l_y (s_3 s_4)_{\sigma_y})_{j_y} \right]_{j_{xy}} l_z \right\}_J \end{aligned}$$

where t_i and s_i are the isospin and spin of the individual particles and (T, J) are respectively the total isospin and angular momentum of the four-body system. Each component F_{α} is thus labelled by a set of 12 quantum numbers to which the symmetry properties of the wavefunction impose the additional constraints: $(-1)^{\sigma_x + \tau_x + l_x} = \varepsilon$ for K and $(-1)^{\sigma_x + \tau_x + l_x} = (-1)^{\sigma_y + \tau_y + l_y} = \varepsilon$ for H.

The boundary conditions are implemented by imposing at large enough values of z the asymptotic behaviour of the solutions. Indeed, at large values of z and for energies below the first inelastic threshold, the solution of equation (1) factorizes into a bound state solution of the 3N Faddeev equations and a plane wave in the z direction whereas the solution of (2) vanishes. The scattering observables are directly extracted from the logarithmic derivative of the K amplitude in the asymptotic region. The asymptotic 3N state is calculated with the same numerical scheme than the one used to solve the 4-body problem. In this case the factorization property, valid only in cartesian coordinates, is an exact numerical property and provides a strong stability test [1].

The numerical methods used are based on the Hermite spline expansion, orthogonal collocation and iterative procedures for solving the linear system. An important step in its solution is done by means of the tensor trick [2].

3. RESULTS

3.1. S-wave approximation

We first discuss the results obtained in the S-wave approximation. By this we mean i) an interaction which acts only on S-wave and ii) that all the angular momenta variables in expansion (3) are zero. The interaction considered is MT I-III with parameters given in [3], slightly different from those of its original version [4]. The scattering length and effective range in different (T,S) channels are given in Table 1 and compared with the existing calculations. Our scattering length values are in agreement with [6] except in the S=1 T=0 channel. For T=1 they are also close to those obtained in [5] although with the original potential parameters.

3.2. n-t scattering with realistic interactions

A step behind the S-wave approximation has been achieved by considering the n-t scattering with realistic NN potentials. This reaction is a pure T=1 channel, free from

Table 1

Low energy N-NNN parameters

T	S	a	r_0	a [6]
0	0	14.75	6.75	*14.7
0	1	3.25	1.82	*2.8
1	0	4.13	2.01	4.0
1	1	3.73	1.87	3.6

Table 2

 ^4He binding energy and r.m.s. radius

	^4He	[2]	[7]	$^4\text{He}^*$
B	30.30	30.31	30.29	0.27
r.m.s.	1.44	-	-	4.99

Table 3

Experimental n-t scattering lengths (in fm.).

a_0	a_1
3.91 ± 0.12	3.6 ± 0.1 [12]
4.98 ± 0.29	3.13 ± 0.11 [13]
2.10 ± 0.31	4.05 ± 0.09 [13]
4.453 ± 0.10	3.325 ± 0.016 [14]

Table 4

Calculated n-t scattering lengths, zero-energy cross section and ^4He binding energy (B_4) and r.m.s. radius (units are fm and MeV)

	a_{0+}	a_{1+}	$\sigma(0)$	B_4	r.m.s.
AV14	4.31	3.79	193.7	23.34	1.56
Nijm II	4.31	3.76	191.6	23.39	1.54
MT I-III	4.10	3.63	177.0		
exp			170.2 ± 9.7		

the difficulties related to the Coulomb interaction and for which accurate low energy scattering data exist [11]. The extrapolated zero energy cross section is $\sigma(0) = 170.2 \pm 9.7$ fm² but the spin-dependent scattering length values are still controverted (see Table 3).

We have calculated the scattering length of the singlet 0^+ and triplet 1^+ states with the AV14 [9] and Nijmegen II [10] potentials. The interaction is limited to the $^1S_0, ^3S_1, ^3D_1$ waves and the FY expansion includes all the components with $l_z = 0, 1$ and $l_y = 0, 1, 2$. The results are displayed in Table 4 together with the ^4He binding energy (B_4) and r.m.s. radius. They are close to those presented in this Conference by the Pisa group [15] and our B_4 value for AV14 shows a relative agreement of 10^{-4} with [7].

The MT I-III results presented in the previous section give a slightly overestimated value for the zero energy cross section $\sigma(0) = 184.6$ fm². However the inclusion of higher partial waves in the FY expansion reduces this value to $\sigma(0) = 177.0$ fm² in closer agreement with experiment. The corresponding scattering lengths are then $a_0 = 4.10$, $a_1 = 3.63$ fm.

The conclusion is that the considered realistic interactions on one hand give results very close to each other and on another hand fail in describing the zero energy n-t scattering as they failed in the 3N case. These problems are strongly correlated and can be solved by including a three-nucleon interaction (TNI). We have considered a simple phenomenological TNI model

$$W(\rho) = W_r \frac{e^{-2\mu\rho}}{\rho} - W_a \frac{e^{-\mu\rho}}{\rho} \quad \rho = \sqrt{x^2 + y^2}$$

and get the overall agreement $B_3 = 8.48$ MeV, $B_4 = 28.56$ MeV, $a_0 = 4.0$ fm, $a_1 = 3.6$ fm and $\sigma(0) = 172.4$ fm² with the parameters $W_r = 500$ MeV, $W_a = 174$ MeV and $\mu = 2.0$ fm⁻¹. This set of values is far from being unique and the only serious constraint is the existence of a repulsive short range part.

The calculations have been completed up to the first 3-body break-up threshold by including the first negative parity states $J^\pi = 0^-, 1^-, 2^-$ corresponding to a n-t relative P-waves. This region ($T_{lab} \approx 3$ MeV) is of particular interest due to the observed structure which has been interpreted as a series of P-waves resonances [16].

Let us first consider the results obtained with MT I-III, displayed in Figure 1. This model conserves separately L and S and consequently the $J^\pi = 0^-, 1^-, 2^-$ states coming from $S=1$ are degenerated. The corresponding cross sections differ only by statistical factors. The remaining $J^\pi = 1^-$ state comes from an $(S=0, L=1)$ coupling. Despite the simplicity of this model the agreement with experimental data, specially in the resonance region, is very good. Only the zero energy region is a little overestimated. The n-t P-waves resonances are thus generated by S-wave NN interaction alone. The effective 1+3 potential is essentially due to the exchange mechanism between the four nucleons. We notice however that a first attempt in describing this cross section with $l_z = 1$ in the K components but keeping zero all the remaining angular momenta in (3) failed.

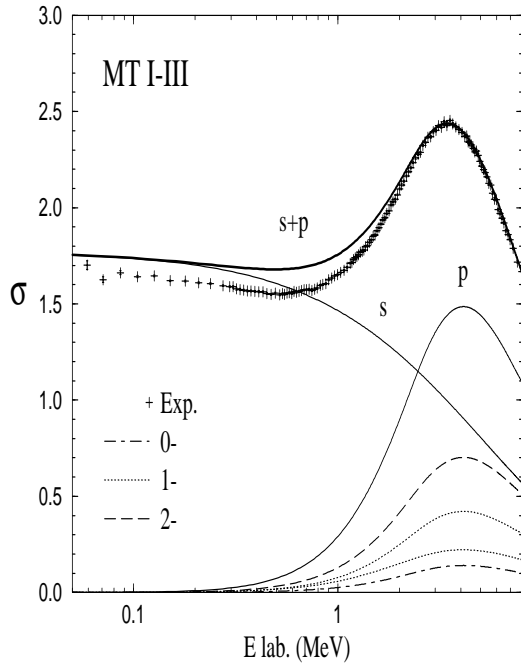


Figure 1. n-t cross section (MT I-III).

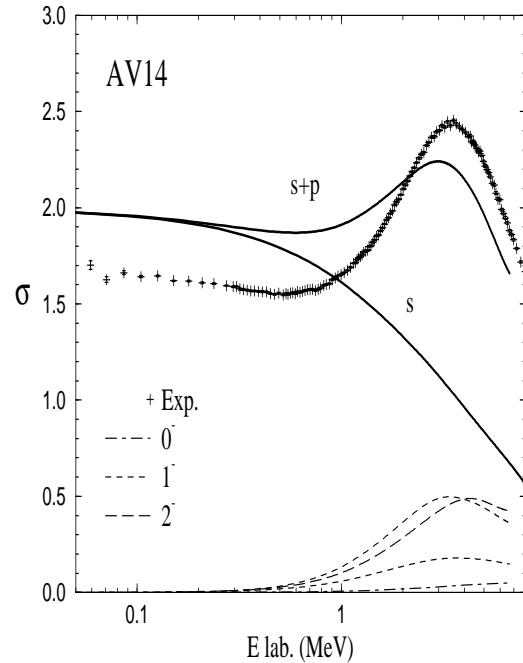


Figure 2. n-t cross section (AV14).

The results obtained with the AV14 are plotted in Figure 2. They are not able to reproduce the experimental data neither in the S-wave nor in the resonance region. Correcting the zero energy with our TNI model results in lowering the value of the cross sections and the disagreement becomes worse. When compared to the MT I-III results one sees that, due to non central terms in the NN potential, the different J^π contributions are shifted. As a consequence the coherent enhancement is weakened and the resonance pick broadened. On another hand the 2^- partial cross section, which should provide the dominant contribution, is noticeably smaller. Trying to account for this disagreement we have included the $^1P_0, ^1P_1, ^3P_1$ waves in V_{NN} , closing the $J=1$ shell. The results, displayed

Table 5

P-wave n-t phase shifts at E=3 MeV (degrees).

	V_{NN} partial waves	0^-	1^-	1^-	2^-
AV14	$^1S_0, ^3S_1, ^3D_1$	21.2	26.6	47.8	35.4
	$^1S_0, ^3S_1, ^3D_1 + ^1P_0, ^1P_1, ^3P_1$	24.1	20.0	37.5	32.7
MT I-III	$^1S_0, ^3S_1$	44.1	30.3	44.1	44.1

in Table 5, show that the corrections go in the bad direction. On the contrary if the $^3S_1 - ^3D_1$ coupling is decreased and $V_{^3S_1}$ enhanced, keeping constant the triton binding energy, the experimental values are reproduced.

The results obtained with MT I-III show the difficulty to understand these resonances in terms of the NN interaction alone. Several reasons can be put forward to explain the apparent difficulty for realistic interactions in describing the n-t scattering before having to conclude about its definite failure. Complementary calculations would be suitable with increasing number of FY components, by adding higher partial waves in the potential, by including realistic TNI, etc. They are however in the limit of the capacities of the best existing computers.

On another hand it is intriguing that very simple interactions work so well. Is it only a chance that the conjugated actions of very sophisticated forces $V_{2N} + V_{3N} + \dots$ result in a very simple two-body effective potential ?. Or should that rather be an indication that simple effective potentials must be introduced from the very beginning in these low energy few-nucleons physics ?.

Acknowledgements. The numerical part of this work has been done with the T3E of the CGCV from C.E.A. and IDRIS from C.N.R.S.

REFERENCES

1. F. Ciesielski, Thèse Université de Grenoble (1997).
2. N.W. Schellingerhout, J.J. Schut, L.P. Kok, Phys. Rev. C 46 (1992) 1192.
3. G.L. Payne, J.L. Friar, B.F. Gibson, Phys. Rev. C 26 (1982) 1385.
4. R.A. Malfliet, J. A. Tjon, Nucl. Phys. A 127 (1969) 161.
5. J.A. Tjon, Phys. Lett. 63B (1976) 391.
6. S.L. Yakovlev, I.N. Filikin, Physics of Atomic Nuclei 58 (1995) 754 and (*) Poster contribution to this Conference.
7. H. Kamada, W. Glockle, Nucl. Phys. A 548 (1992) 205-226.
8. J.A. Tjon, Phys. Lett. 56B (1975) 217.
9. R.B. Wiringa, R.A. Smith and T.L. Ainsworth, Phys. Rev. C 29 (1984) 1207.
10. V.G. Stoks, R.A. Klomp, C.P. Terheggen, J.J. de Swart, Phys. Rev. C49(1994) 2950.
11. T.W. Phillips, B.L. Berman, J.D. Seagrave, Phys. Rev. C 22 (1980) 384.
12. T.W. Phillips, B.L. Berman, J.D. Seagrave, Phys. Lett. 91B (1980) 200.
13. H. Rauch et al., Phys. Lett. 165B (1985) 39.
14. G.M. Hale et al., Phys. Rev. C 42 (1990) 438.
15. M. Viviani, Contribution to this Conference.
16. D.R. Tilley, H.R. Weller, G.M. Hale, Nucl. Phys. A541 (1992) 1.